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Financing Losers in Competitive Markets*

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Projects with negative expected value cannot obtain financing in competitive capital markets if all potential investors are risk neutral and have identical beliefs about the distribution of the project's net revenue. We present a series of examples with heterogeneous beliefs in which it is possible for a project to obtain financing even though all investors in the project believe, conditional on the project being undertaken, that the project has negative expected value. An important feature of the examples is that the differences in beliefs are due only to differences in information, and are not simply arbitrary unexplained differences in opinions. *Journal of Economic Literature Classification Numbers:* D8, G1.

I. INTRODUCTION

Projects are financed in competitive capital markets by selling securities to investors. Because securities are claims on a project's net revenue, risk-neutral investors will not buy the securities of projects that have

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negative expected net revenue. This result—that projects with negative expected value cannot obtain financing—holds only if all potential investors have identical subjective distributions for the project's net revenues. If investors have different subjective distributions for net revenues, then it is possible for a project to obtain financing even though all investors in the project believe, conditional on the project being undertaken, that the project has a negative expected value.

In this paper we develop an information-theoretic model of project financing and present examples in which all investors believe that a project has a negative expected value if undertaken, and yet the project is financed by these investors. While it is crucial that investors have heterogeneous beliefs, it is noteworthy that in this framework the differences in beliefs are due only to differences in information and are not simply arbitrary unexplained differences in opinions. Investors receive different pieces of information and are sophisticated in their use of this information to obtain Bayesian posteriors. If the investors could credibly pool their information, they would agree on the distribution of the project's net revenues. However, in our model there is no mechanism by which investors can pool their information, either directly or indirectly through information-revealing behavior. Thus, the differences in beliefs persist in the security market equilibrium, which makes it possible for projects with negative expected value to obtain financing.

The model we develop is best viewed as a model of initial public offerings. An entrepreneur has access to a project with uncertain net revenues and must obtain external financing. The entrepreneur offers a group of securities for sale. The prices of these securities, as well as their payoff characteristics, are set by the entrepreneur. Investors, who are each so small that the capital market is competitive, decide which, if any, securities to subscribe to. If investors believe that a security is priced too high, they will not subscribe to the security, and the project does not proceed. If investors think that the security is priced correctly, or even underpriced, then they will subscribe to the security. Because the price of the security is set before investors act in the capital market, the price of the securities cannot reveal information possessed by investors.

Why will investors subscribe to securities of a project that they think has a negative expected value? The answer is that although securities are claims on a project's net revenues, the expected value of subscribing to a security can differ from the expected value of the project's net revenue. We explore two sources of a wedge between the expected value of a project and the expected value of subscribing to its securities. First, if a security is oversubscribed, the available securities are allocated to subscribers according to a random rationing scheme. Because the probability of acquiring a security after subscribing to it differs in different contingencies, there is a wedge between the expected value of subscribing to a security and the expected value of the project's net revenues, even if there is only one type of security. We present an example with one type of security in which investors not only believe that the project has negative value, but also know that all the other investors believe that the project has a negative value. It cannot be common knowledge, however, that the project (when undertaken) has negative value.

A second wedge arises if a project is financed by two or more different types of securities. Although the bundle of all securities is a claim on the project's net revenues, different types of securities are different state-contingent claims on a project's net revenues. With heterogeneous beliefs, different investors may subscribe to different securities and thus finance the project even though no single investor would want to hold a bundle of all of the project's securities.

There are other examples in the literature in which decisions taken on the basis of private information lead to seemingly suboptimal or non-equilibrium outcomes. For example, stock market prices may not reflect fundamentals. Allen *et al.* (1993) have constructed examples in which all traders know that the price of a security will fall in the future and yet the investors still wish to hold or buy the security. Jackson and Peck (1992) show that bubbles can result from private information that is unconnected to market fundamentals.

There are also examples with sequential decision making in which agents ignore their own private information. Welch (1992) illustrates how an overpriced initial public offering can succeed, even though if all the investors' private information were public it would fail. When investors observe that the offering is going well, they infer that earlier investors did not believe the offering was overpriced; then they may also be willing to invest. But since later investors only observed that the offering is going well, and not how many of the investors believed that the offering was not overpriced, they may incorrectly conclude that the offering is not overpriced. Banerjee (1992) and Bikhchandani *et al.* (1992) also study models of cascades or fads: the pattern of private information and the sequencing of observed decisions can lead to agents ignoring their own private information. In our work, by contrast, all decisions are simultaneous and no investor is updating on the basis of the information or action of any other investor.

More closely related to the current paper is Sebenius and Geanakoplos (1983), who take as their starting point the observation that if two people have different probability assessments about the realization of an uncertain event, they can design a contingent agreement, such as a gamble, that offers each of them a positive expected value. They show that, for any possible contingent agreement, it cannot be common knowledge that both sides wish to accept that agreement. Consistent with that paper, as well as Geanakoplos and Polemarchakis (1982), in our model it will not be common knowledge that all investors wish to invest and there is no repeated exchange of information on the willingness to enter an agreement. Our examples are also consistent with the no-trade theorem of Milgrom and Stokey (1982). This theorem states that if traders have achieved an ex ante efficient allocation, then the arrival of new (private) information cannot by itself generate new trades. In our model private information, *per se*, is not the source of security purchases, because the projects have positive ex ante value.

II. AN INFORMATION THEORETIC MODEL

A. *The Project and the Securities*

Consider a risk-neutral entrepreneur who wants to raise funds to finance a project with a publicly known cost C and unknown revenue $x \geq 0$. We let S denote the finite space of uncertainty, and refer to elements of S as states of the world, or states. The collection of possible values of the project's revenue x is given by $X \equiv \{x_s \in \mathcal{R}_+ : s \in S\}$.[1] Let $\rho(s)$ be the probability that the state is s, so that ρ is the prior distribution on S. We assume $\rho(s) > 0$ for all $s \in S$.

The entrepreneur has no funds of her own to invest and therefore must raise funds by selling securities.[2] The entrepreneur offers N types of securities for sale. Security 1 is a residual claim on the project's net revenues and thus is equity. The entrepreneur retains a fraction $\sigma \geq 0$ of the firm's equity, and the remaining fraction $(1 - \sigma)$ is sold to external investors. The entrepreneur does not hold any of the other securities; they are sold entirely to external investors. In addition to an equity ownership in the project, the entrepreneur receives a lump-sum payment, $\pi \geq 0$ (a supernormal salary for example).[3]

Each security is characterized by a price and a payoff function. A unit (Lebesgue) measure of each security is offered for sale to external investors. The total amount of capital raised by selling security n to external

[1] Without loss of generality, x_s can be interpreted as the expected value of a return that is drawn from some distribution F_s. In Section VI, we associate a distribution over returns to each state s, and calculate project and security values (introduced below) explicitly.

[2] The entrepreneur offers securities through a risk-neutral investment bank. Rather than study the relationship between the entrepreneur and the investment bank, we will treat the entrepreneur and the investment bank as a single entity with access to the same information.

[3] Nothing depends on σ, $\pi \neq 0$. Allowing the entrepreneur to take a share of the project's revenues only makes it harder to construct examples in which projects that all investors believe have negative value are financed.

investors is p_n. Since the entrepreneur retains a fraction σ of the project's equity, p_1 is the amount paid by external investors for $(1 - \sigma)$ of the project's equity (security 1). Since all of the remaining securities are sold entirely to external investors, p_n is the total amount paid by external investors for all of security n, $n = 2, \ldots, N$.

Each security specifies a payment to the owners of the security as a function of the state. In particular, $y_n(s)$, $n = 1, \ldots, N$, is the amount received by the owners of security n in state s. Because external investors own only a fraction $(1 - \sigma)$ of the first security, they pay p_1 for this security and receive a payoff $(1 - \sigma)y_1(s)$. For the remaining securities, external investors pay p_n and receive $y_n(s)$.

In order for the project to be undertaken, the funds raised by the sale of securities, $\Sigma_n p_n$, must be sufficient to cover the known cost of the project, C, plus the fixed payment to the entrepreneur, π:

$$\Sigma_n p_n - C - \pi \geq 0. \tag{1}$$

After the project's revenue, x_s, is realized, the project has funds equal to $x_s + \Sigma_n p_n - C - \pi$. These funds are distributed to the owners of the securities according to the payoff functions y_n. Since all of the available funds are distributed, we have

$$\Sigma_n y_n(s) = x_s + \Sigma_n p_n - C - \pi \geq 0. \tag{2}$$

Equation (1) and the assumption that $x_s \geq 0$ together imply that the amount of funds available on the right hand side of Eq. (2) is nonnegative.

We require that the payoff functions satisfy

$$y_n(s) \geq 0, \qquad \text{for all } s \in S,\ n = 1, \ldots, N. \tag{3}$$

This restriction corresponds to limited liability: no security owner can be forced to pay an additional amount after purchasing the security. Since the amount of funds available is nonnegative, this is without loss of generality.

B. *The Valuation of Securities*

Because all investors are risk neutral, the value of buying any security is the expected value of its payoff minus the amount paid for the security. In state s, the project's revenue is x_s and the payoff to external owners of security 1 is $(1 - \sigma)y_1(s)$. Since external investors pay p_1 in aggregate for security 1 (equity), the net value of buying this security is

$$v_1(s) = (1 - \sigma)y_1(s) - p_1. \tag{4}$$

External investors receive all of the payoffs to securities 2, . . . , N, because the entrepreneur does not retain any of these securities. The net value to external investors of buying security n in state s is

$$v_n(s) = y_n(s) - p_n, \qquad n = 2, \ldots, N. \tag{5}$$

The entrepreneur's expected net benefits come from two sources. First, the entrepreneur may retain a fraction σ of security 1; second, the entrepreneur may receive a supernormal salary π. In state s, the entrepreneur's net benefit, $v_{\mathrm{E}}(s)$, is

$$v_{\mathrm{E}}(s) = \sigma y_1(s) + \pi. \tag{6}$$

The entrepreneur is endowed with sole access to the project. Because the entrepreneur invests no resources of her own in the project, $v_{\mathrm{E}}(s) \geq 0$ (this is immediate from Eq. (3) and the assumption that $\pi \geq 0$).

Equations (4)–(6) give the values of all of the claims on the project. We can relate the values of these claims to the net revenue of the project, $x_s - C$, as follows,

$$v_{\mathrm{E}}(s) + \Sigma_n v_n(s) = \pi + \Sigma_n [y_n(s) - p_n] = x_s - C, \tag{7}$$

where the first inequality is obtained by adding Eqs. (4), (5), and (6); the second equality follows from Eq. (2). Equation (7) indicates that in any state s the value of the project, $x_s - C$, equals the total valuation of all of the securities held by external investors plus the value of the claims held by the entrepreneur.

C. *Complete and Incomplete Subscription*

When the entrepreneur offers securities for sale, each investor knows the prices and payoff functions of the securities and, based on her subjective distribution for s, decides which of the securities to subscribe to. An investor may subscribe to no securities, one security, or many types of securities. *Incomplete subscription* occurs when there is at least one security for which there are insufficient subscribers. In case of incomplete subscription, funds are not collected from investors and the project does not proceed. *Complete subscription* occurs when all securities have sufficient subscribers. If there is complete subscription and a security is oversubscribed, then it must be rationed. We use a random rationing scheme in which each subscriber to a security receives one unit of that security with a probability equal to the reciprocal of the measure of investors subscribing to that security.[4] For example, if the measure of subscribers to a security is 2, then each subscriber has a 50% chance of getting the security because the measure of that security offered for sale is 1.[5]

D. *Information Structure*

Assume that the investors can be assigned to M classes of equal size. Specifically, a typical investor class, denoted by i, is a continuum of unit (Lebesgue) measure of investors. All investors in a class are identical and have identical information.

To illustrate the role of the information possessed by different classes, as well as to illustrate some of the definitions that follow, we begin with an example (Example 1). In Example 1, $M = 2$, so there are two classes of investors, denoted as class 1 and class 2. There are nine possible states, $S = \{s_{ij}: i, j = 1, 2, 3\}$, each equally likely. The information structure is illustrated in Table IA. Investors in class 1 only learn which row contains the true (realized) state and investors in class 2 only learn which column contains the true state. Thus, for instance, if the state is s_{23}, class 1 investors learn that the state is in row 2 and class 2 investors learn that the state is in column 3.

The information structure of investor class i is described formally as a partition of S, written P_i. In Table IA, the partition of investor class 1 has three elements: $P_1 = \{\{s_{11}, s_{12}, s_{13}\}, \{s_{21}, s_{22}, s_{23}\}, \{s_{31}, s_{32}, s_{33}\}\}$; the partition of investor class 2 also has three elements: $P_2 = \{\{s_{11}, s_{21}, s_{31}\}, \{s_{12}, s_{22}, s_{32}\}, \{s_{13}, s_{23}, s_{33}\}\}$. Each investor receives a private signal that indicates

[4] Since investors are risk neutral, if a security has a positive expected value then the demand for that security is unbounded. Strictly speaking, rationing must occur whenever a security has a positive value. The assumption that if an investor receives any portion of a security, then she receives one unit is an assumption about rationing. However, we will reserve the term "rationing" to describe a situation in which the measure of investors subscribing to a security exceeds one, the measure of the security supplied.

Assuming that investors either receive zero or one unit of a security is a normalization. Since we assume the space of investors is an atomless measure space (so that each investor is an infinitesimal fraction of the population), no single investor (or finite collection) can completely subscribe to a security. Nothing is affected by setting an arbitrary *finite* bound on the security sale to any investor (this bound need not be uniform in the investor).

[5] An equivalent nonrandom rationing scheme in our setting is that each subscriber receives an amount equal to the reciprocal of the measure of subscribers to that security. Since investors are risk-neutral, receiving one unit of a security with some probability is equivalent to receiving that proportion of one unit with certainty. Note that if investors were allowed to revise their subscription after observing how much of each security they were able to buy, the two rationing schemes would not be equivalent. The investor would have more information under the alternative nonrandom scheme than under the random rationing scheme we use in the text. This nonrandom rationing scheme is consistent with the scheme described in the previous footnote.

TABLE IA

	Class 2 investors learn column				Prior probability, ρ		
Class 1	s_{11}	s_{12}	s_{13}		1/9	1/9	1/9
investors	s_{21}	s_{22}	s_{23}		1/9	1/9	1/9
learn row	s_{31}	s_{32}	s_{33}		1/9	1/9	1/9

which element of the investor's partition contains the true state, s. We let $t_i(s)$ be the element of partition P_i that contains the state s. Using this formalism, we can describe the information structure when the state is s_{23} as $t_1(s_{23}) = \{s_{21}, s_{22}, s_{23}\}$ and $t_2(s_{23}) = \{s_{13}, s_{23}, s_{33}\}$. We can interpret $t_i(s)$ as a private signal that investors in class i receive. The structure of revenues can be interpreted as the outcome of a publicly observed signal.

We use the terms ex ante, interim, and ex post to describe different information scenarios (not different times). *Ex ante* refers to the scenario in which all investors know only the prior distribution $\rho(s)$ over S. *Interim* refers to the scenario in which each investor has received the signal that identifies which element of her partition contains the true state. *Ex post* refers to the scenario in which the true state s, and so the revenue x_s, is known.

We have already discussed the ex ante scenario in which investors know the prior distribution $\rho(s)$ over S, and we have discussed the ex post scenario in which the state is known. The interim scenario requires further elaboration. If the true state is s_{23}, investors in class 1 know the state is in $\{s_{21}, s_{22}, s_{23}\}$, and update their priors to obtain an interim posterior that assigns a probability of $\frac{1}{3}$ to each state in $\{s_{21}, s_{22}, s_{23}\}$, and zero probability to the other states. This interim posterior depends both on the investor class and the true state. Formally, when the true state is s', investors in class i know that the state is contained in $t_i(s')$ and update their priors to the *interim posterior distribution*, $\rho(s|t_i(s'))$, where

$$\rho(s|t_i(s')) \equiv \begin{cases} \rho(s)/\sum_{s''\in t_i(s')} \rho(s''), & \text{if } s \in t_i(s'), \\ 0, & \text{if } s \notin t_i(s'). \end{cases} \tag{8}$$

E. *Investors' Decisions and Equilibrium*

In deciding whether to bid for security n, an investor takes account of the rationing scheme, which specifies the investor's probability of actually being able to purchase a security for which she submits a bid. The probability of obtaining a security depends on how many other investors bid for that security, so the investor must take account of the equilibrium number of bids for security n in deciding whether to submit a bid for that security. To formally describe the role in the investor's decision of the equilibrium number of bids, we introduce three concepts: (1) the security–demand correspondence; (2) the market demand for each security; and (3) the set of states for which there is complete subscription.

1. Security–Demand Correspondence. If the state is s, investors in class i receive a signal that the state is in $t_i(s)$. Based on this information, investors in class i choose which, if any, securities to subscribe to. The security–demand correspondence, $\xi(s, i)$, describes the set of securities that investors in class i subscribe to when the true (but unknown) state is s. Formally, $\xi: S \times M \rightarrow \mathcal{P}(N)$, where $\mathcal{P}(N)$ is the power set of N.[6] We require that the security–demand correspondence respect the information structure in the sense that if class i's partition pools s with s' ($s' \in t_i(s)$), then $\xi(s', i) = \xi(s, i)$, i.e., it is measurable with respect to the investors' information partitions.

Table IB shows a security–demand correspondence for a security offer with one security for Example 1. The matrix on the left shows the securities that investors in class 1 subscribe to, and the matrix on the right shows the securities that investors in class 2 subscribe to. For example, if the true state is s_{22}, then investors in both classes subscribe to security 1, while only investors in class 1 subscribe to the security if the true state is s_{13}.

2. The Market Demand Function. The market demand function for security n, $Q_n(s, \xi)$, indicates the measure of investors (i.e., the "number" of investors) that subscribe to security n. The measure of investors that subscribe to security n depends on the signal received by each investor class, which depends on the true state, and on the security–demand correspondence, ξ. In equilibrium, all investors in a class choose the same set of securities, and $Q_n(s, \xi)$ will be the number of investor classes that subscribe to security n. Table ID, below, shows the market demand function for Table IB.

3. Complete Subscription. An investor's subscription to a security can be satisfied only in the case of complete subscription. Thus, it is

[6] The power set of N is the collection of all subsets of N. Strictly speaking, since different investors in the same class may choose different securities, it would be more accurate to represent security demands for a class i and a state s by a vector $(z(N'))_{N' \subset N}$ satisfying $z(\cdot) \geq 0$ and $\sum_{N' \subset N} z(N') = 1$ (the proportion of investors choosing the set of securities N' is $z(N')$). The security–demand correspondence described in the text assumes that all investors in a class choose the same set of securities. The only situation in which a security can be subscribed to by some, but not all, investors in a class is when the security has a zero value. The more restrictive formulation in the text suffices for our purposes.

TABLE IB
A SECURITY–DEMAND CORRESPONDENCE FOR A SECURITY OFFER WITH ONE SECURITY

Investor class 1
$\xi(s, 1)$

{1}	{1}	{1}
{1}	{1}	{1}
∅	∅	∅

Investor class 2
$\xi(s, 2)$

{1}	{1}	∅
{1}	{1}	∅
{1}	{1}	∅

important for investors to know when there will be complete subscription and when there will be incomplete subscription. Let $D(\xi)$ be the set of states for which there is complete subscription when investor behavior is described by the security–demand correspondence ξ. Formally, $D(\xi) \equiv \{s \in S: \forall n, \exists i \text{ s.t. } n \in \xi(s, i)\}$. We often suppress the dependence of D on ξ for notational convenience. For the security–demand correspondence in Table IB, $D(\xi) = \{s_{11}, s_{12}, s_{13}, s_{21}, s_{22}, s_{23}, s_{31}, s_{32}\}$. The state s_{33} is not in $D(\xi)$, because neither class of investor subscribes to the security in that state.

F. The Value of Subscribing to a Security

When the state is s', investors in class i learn that the state is in $t_i(s')$, and they know that the project will proceed only if the state is in $t_i(s') \cap D$. The *D-interim posterior* of investors in class i is the posterior distribution over states, where the posterior is based on t_i and is conditional on the project proceeding. The D-interim posterior distribution $\rho(s \mid t_i(s'), D)$ is given by

$$\rho(s \mid t_i(s'), D) \equiv \begin{cases} \rho(s)/\sum_{s'' \in t_i(s') \cap D} \rho(s''), & \text{if } s \in t_i(s') \cap D, \\ 0, & \text{if } s \notin t_i(s') \cap D. \end{cases} \tag{9}$$

The D-interim posterior is used by investors in class i to evaluate the expected value of the project's revenues x, conditional on the project being financed. This conditional expected value will figure prominently in our definitions of efficiency.

What is the expected net value to an investor of subscribing to security n, conditional on the information that the state is in $t_i(s')$? The probability that the state is s is given by the interim posterior $\rho(s \mid t_i(s'))$ in Eq. (8). If s is not in D, the project is not financed and the subscription has zero net value. However, if s is in D, an investor has a probability of $1/Q_n(s, \xi)$ of obtaining security n. If the investor succeeds in obtaining security n, it has a value of $v_n(s)$. Thus, if the state s is in D, the expected value of subscribing to security n is $v_n(s)/Q_n(s, \xi)$. When the expected value of the subscription is weighted by the conditional probability that s is the true state, the expected value to subscribers in class i of subscribing to security n is

$$V_n(t_i(s'), \xi) = \sum_{s \in t_i(s') \cap D} \{v_n(s)/Q_n(s, \xi)\}\rho(s \mid t_i(s')). \tag{10}$$

If the expected value of subscribing to security n is positive, then all investors in class i subscribe to security n; if the expected value of subscribing to security n is negative, none of the investors in class i subscribes to security n. Note that the expected value of subscribing to security n depends on the security–demand correspondence; investors take account of the market demand for a security in deciding whether to subscribe to the security.

G. *Security Market Equilibrium*

Having completed the discussion of investor behavior we now define a competitive security equilibrium.

DEFINITION. A security offer, which specifies π, σ, p_n, and y_n, $n = 1, \ldots, N$, and a security–demand correspondence ξ constitute a *competitive security equilibrium* if for all $s \in S$, for $n = 1, \ldots, N$,

(i) if $V_n(t_i(s), \xi) > 0$, then $n \in \xi(s, i)$ and
(ii) if $V_n(t_i(s), \xi) < 0$, then $n \notin \xi(s, i)$.

Condition (i) states that if the expected value of subscribing to security n is positive for investors in class i, they will subscribe to security n. Condition (ii) states that if the expected value of subscribing to security n is negative for investors in class i, they will not subscribe to security n.

We illustrate in Tables IC, ID, and IE a competitive security equilibrium for Example 1 with one security (equity). The information structure is given by Table IA. We assume that the entrepreneur retains no equity and does not earn a supernormal salary, so that $\sigma = \pi = 0$. Under these

TABLE IC
VALUE OF SECURITY, $v_1(s) = x_s - C$

10	4	−5
4	−10	4
−5	4	10

TABLE ID
MARKET DEMAND FUNCTION FROM TABLE IB, $Q_1(s, \xi)$

2	2	1
2	2	1
1	1	0

TABLE IE
VALUE OF SUBSCRIBING, $v_1(s)/Q_1(s, \xi)$

5	2	−5
2	−5	4
−5	4	—

assumptions, the value of equity equals revenue minus the cost of the project (i.e., $v_1(s) = x_s - C$ for all s). The security–demand correspondence in Table IB, along with the security valuations in Table IC, constitutes a competitive security equilibrium.

Note that if the state is s_{22}, there will be complete subscription and both investors will subscribe to the security, despite the fact that it has a valuation of -10. Why do both investors subscribe to the security in this case? The answer is that the rationing scheme indicates that if the state is s_{22}, a subscriber has only a 50% chance of obtaining the security. Table IE lists the value of subscribing to the security, which takes into account the rationing implied by the market demand in Table ID. Note that the expected value of subscribing to the security is positive ($\frac{4}{3}$) for each class of investor when the state is s_{22}. There is incomplete subscription in s_{33}, indicated by the dash, and so the project is not undertaken in that state.

The rationing in this example may appear to be perverse because the security is rationed when the state is s_{22} and the expected value of the project is negative (-10). This appearance is deceiving, however, since the expected value of the security, conditional on its being rationed (i.e., $s \in \{s_{11}, s_{12}, s_{21}, s_{22}\}$), is positive (2), while the expected value of the security, conditional on its not being rationed and the project's being financed ($s \in \{s_{13}, s_{23}, s_{31}, s_{32}\}$), is negative ($-\frac{1}{2}$).

III. NOTIONS OF EFFICIENCY

The general question motivating this research is whether a competitive security equilibrium does an effective job of choosing the appropriate projects to finance. Does it allow projects with positive expected net revenues to obtain financing, while at the same time preventing projects with negative net expected values from obtaining financing?[7] To pose this question more sharply, we need to introduce various concepts of efficiency.

A project is said to be *ex post inefficient at s* if

$$x_s - C < 0. \tag{11}$$

A project that is not ex post inefficient at s is said to be *ex post efficient at s*. A competitive security equilibrium is said to be *ex post efficient* when $s \in D$ if and only if the project is ex post efficient at s.

[7] Our definitions of efficiency rely on the risk neutrality of the agents. Projects with negative expected value will have some insurance value if the payoffs to the project are negatively correlated with the market portfolio and if some agents are risk averse.

If the project's revenue were common knowledge, then projects would be financed and undertaken only in states in which they were ex post efficient. Because the project's revenue is not common knowledge, we need to introduce alternative definitions of inefficiency to incorporate the information structure of the model. We will not be concerned with the inefficiency of the project *per se*, but rather with the inefficiency of the security equilibrium.

A security equilibrium is said to be ex ante inefficient if, conditioned only on the fact that the project is undertaken, the expected value of revenue minus cost is negative. Formally, a security equilibrium is *ex ante inefficient* if

$$\sum_{s \in D} (x_s - C)(\rho(s)/[\Sigma_{s' \in D}\, \rho(s')]) < 0. \tag{12}$$

A security equilibrium is *ex ante efficient* if it is not ex ante inefficient according to Eq. (12).

We can also evaluate the efficiency of a security equilibrium conditional on information received by investors. A security equilibrium is said to be D-interim inefficient at s according to investors in class i if the conditional expected value of net revenue is negative. In forming the conditional expectation, investors in class i use their information, $t_i(s)$, and condition on the project being undertaken. Formally, a security equilibrium is *D-interim inefficient at s according to investors in class i* if

$$\sum_{s' \in t_i(s) \cap D} (x_{s'} - C)\rho(s' \mid t_i(s), D) < 0. \tag{13}$$

If investors in class i view the security equilibrium as D-interim inefficient, then based on their available information, they think that when the project is undertaken, it will on average incur losses.

The notions of ex ante inefficiency and ex post inefficiency are unambiguous, in that all classes of investors agree on the questions of whether a security equilibrium is ex ante inefficient and whether it is ex post inefficient. By contrast, the notion of D-interim inefficiency is investor-dependent and signal-dependent: for a given state s some investors may view the security equilibrium as D-interim inefficient while other investors do not; also, a given investor may view a particular security equilibrium as D-interim inefficient or not depending on the signal, $t_i(s)$, she receives.

The various definitions of inefficiency involve $x_s - C$. Alternatively, we can express these definitions in terms of the valuations $v_E(s)$ and $v_n(s)$, $n = 1, \ldots, N$. Recall from Eq. (7) that $x_s - C = v_E(s) + \Sigma_n v_n(s)$, so we can replace $x_s - C$ by $v_E(s) + \Sigma_n v_n(s)$ in the definitions of inefficiency.

Before examining various security equilibria, we state the following theorem, which is proved in the Appendix.

THEOREM 1. *Competitive security equilibria are ex ante efficient.*

This theorem formalizes the notion that only projects with ex ante nonnegative expected net revenue will be able to obtain financing from rational investors.

However, as we now show using Example 1, rational investors can provide financing in equilibria that are *D*-interim inefficient. The equilibrium is *D*-interim inefficient according to both classes when the state is s_{22}. Recall that the value of the project is equal to the value of the security. Both classes are willing to subscribe to the security because, due to rationing, the value of *subscribing* to the security is positive ($\frac{1}{3}$) for each class while the expected value of the security is negative ($-\frac{2}{3}$) for each class. The equilibrium is ex ante efficient, however, having an expected value (conditional on financing) of $\frac{3}{4}$.

Although the project is financed when the state is s_{22}, we cannot conclude that there is overinvestment. When the state is s_{33}, the project has a positive expected value according to the interim posterior of both investors and yet the project in that case is not financed.

IV. AN EQUILIBRIUM KNOWN TO BE *D*-INTERIM INEFFICIENT BY ALL INVESTORS

We have just shown that it is possible for a project to be completely subscribed in a competitive security equilibrium even though all investors believe the equilibrium is *D*-interim inefficient. In this section we present an example that goes a step further: a competitive security equilibrium in which there is complete subscription when every investor believes the equilibrium is D-interim inefficient, and every investor knows that every other investor also believes the equilibrium is D-interim inefficient.

There are six equiprobable states, $\{s_{11}, s_{13}, s_{21}, s_{22}, s_{32}, s_{33}\}$. Investors in class 1 have the partition $P_1 \equiv \{\{s_{11}, s_{13}\}, \{s_{21}, s_{22}\}, \{s_{32}, s_{33}\}\}$ and investors

TABLE IIA

	Class 2 investors learn column				Prior probability, p		
Class 1	s_{11}		s_{13}		1/6		1/6
investors	s_{21}	s_{22}			1/6	1/6	
learn row		s_{32}	s_{33}			1/6	1/6

TABLE IIB
SECURITY-DEMAND CORRESPONDENCE

Investor class 1
$\xi(s, 1)$

$\varnothing$		$\varnothing$
{1}	{1}	
	{1}	{1}

Investor class 2
$\xi(s, 2)$

{1}		{1}
{1}	$\varnothing$	
	$\varnothing$	{1}

TABLE IIC
VALUE OF SECURITY, $v_1(s) = x_s - C$

4		−5
−6	4	
	−5	11

TABLE IID
MARKET DEMAND FUNCTION FROM TABLE IIB, $Q_1(s, \xi)$

1		1
2	1	
	1	2

TABLE IIE
VALUE OF SUBSCRIBING, $v_1(s)/Q_1(s, \xi)$

4		−5
−3	4	
	−5	$5\frac{1}{2}$

in class 2 have the partition $P_2 \equiv \{\{s_{11}, s_{21}\}, \{s_{22}, s_{32}\}, \{s_{13}, s_{33}\}\}$, illustrated in Table IIA. There is only one security (equity) and $\sigma = \pi = 0$, so that $v_1(s) \equiv x_s - C$. The subscription by investor class 1 ($\xi(s, 1)$), the subscription by investor class 2 ($\xi(s, 2)$), the value of the security ($v_1(s)$), the market demand for the security ($Q_1(s, \xi)$), and the value of subscribing ($v_1(s)/Q_1(s, \xi)$) are shown in Tables IIB–E.

It is straightforward to verify that the security-demand correspondence in Table IIB is part of a competitive security equilibrium.[8] To examine the question of D-interim inefficiency suppose that the state is s_{21} and consider an investor in class 1. Despite the fact that the expected value of the security is negative ($E\{v_1(s) \mid t_1(s_{21})\} = (\frac{1}{2})(-6) + (\frac{1}{2})(4) = -1$), the expected value of subscribing to the security is positive ($V_1(t_1(s_{21}), \xi) = (\frac{1}{2})(-6/2) + (\frac{1}{2})(4) = 1/2$), because the investor will be rationed if the state is s_{21} (note that $Q_1(s_{21}, \xi) = 2$). Thus investors in class 1 subscribe to the security when the true state is s_{21} despite the fact that they believe the equilibrium is D-interim inefficient. Similarly, for an investor in class 2, the expected value of the security is negative ($E\{v_1(s) \mid t_2(s_{21})\} = (\frac{1}{2})(4) + (\frac{1}{2})(-6) = -1$). Thus, both investors believe the equilibrium is D-interim inefficient. Note that, conditional on the security's being rationed ($s \in \{s_{21}, s_{33}\}$), its ex-

[8] The equilibrium is not unique. Another equilibrium is given by class 1 investors only subscribing in row 3 (when $s \in \{s_{32}, s_{33}\}$) and class 2 investors only subscribing in column 3 (when $s \in \{s_{13}, s_{33}\}$).

pected value is positive ($2\frac{1}{2}$), and that conditional on its not being rationed ($s \in \{s_{11}, s_{13}, s_{22}, s_{32}\}$), its expected value is negative ($-\frac{1}{2}$).

Now let us examine what investors think other investors think. When the state is s_{21}, an investor in class 1 knows that the state is either s_{21} or s_{22}. If the state is s_{21}, then (as shown above) investors in class 2 believe the equilibrium is D-interim inefficient. If the state is s_{22}, then investors in class 2 know that the state is either s_{22} or s_{32}. In this case, the expected value of the security is also negative ($E\{v_1(s)|t_2(s_{22})\} = E\{v_1(s)|\{s_{22}, s_{32}\} = (\frac{1}{2})(4) + (\frac{1}{2})(-5) = -\frac{1}{2}$) and investors in class 2 believe the equilibrium is D-interim inefficient. Hence investors in class 1 know that investors in class 2 believe the equilibrium is D-interim inefficient. Similarly, it is easy to check that (again at s_{21}) investors in class 2 know that investors in class 1 believe the equilibrium is D-interim inefficient.

However, it is not common knowledge that the equilibrium is D-interim inefficient.[9] When the state is s_{21}, investors in class 1 cannot rule out the possibility that investors in class 2 think that investors in class 1 believe the equilibrium is D-interim efficient. In particular, investors in class 1 know that investors in class 2 may have learned that the state was in column 2, in which case investors in class 2 cannot rule out class 1 having observed row 3. If investors in class 1 did observe row 3, their conditional expected value of the project would be positive ($E\{v_1(s)|\{s_{32}, s_{33}\} = 3$). This result about common knowledge holds more generally and we state this as our next theorem:

THEOREM 2. *Fix a competitive security equilibrium. It is never common knowledge that the equilibrium is D-interim inefficient.*

Intuitively, if an event is common knowledge (to all the investors) at some state s, then at s, we can redefine the set of all possible states to be this common knowledge event. But then Theorem 1 implies that it cannot be common knowledge that the equilibrium is D-interim inefficient. The formal argument is in the Appendix.

[9] Intuitively, an event is common knowledge if everyone knows the event occurred, everyone knows that everyone knows the event occurred, everyone knows that everyone knows that everyone knows the event occurred and so on. Aumann (1976) introduced the following equivalent definition using information partitions: Denote by $\wedge P$ the finest common coarsening, or *meet*, of $\{P_1, \ldots, P_M\}$; that is, $\wedge P$ is the finest partition of S having the property that, for all i, if $t_i \in P_i$, then there exists $t \in \wedge P$ with $t_i \subseteq t$. The finest common coarsening in all our examples is the trivial partition $\{\{S\}\}$. An event $E \subseteq S$ is *common knowledge at s* if E contains that member of $\wedge P$ containing s. Aumann (1976) proves that agents cannot agree to disagree (i.e., agents cannot have different posteriors that are common knowledge). The importance of common knowledge assumptions is surveyed in Binmore and Brandenburger (1990) and Geanakoplos (1988, 1992).

V. THREE CLASSES OF INVESTORS

In this section, we present an example that avoids two unappealing features of the previous examples: (1) Example 2 depends on a particular information structure that is inconsistent with the common assumption that agents receive independent signals. The information structure in Example 3 below allows the natural interpretation that investors in different classes receive independent signals. (2) Examples 1 and 2 depend upon rationing securities in some states in which the project's net revenue is negative.

In this example (Example 3), there are eight possible states $\{s_{ijk}: i, j, k = 1, 2\}$. There are two securities that will be interpreted as stocks and bonds in the next section. There are three classes of investors. Investors in class 1 have the partition $P_1 \equiv \{\{s_{1jk}: j, k = 1, 2\}, \{s_{2jk}: j, k = 1, 2\}\}$, investors in class 2 have the partition $P_2 \equiv \{\{s_{i1k}: i, k = 1, 2\}, \{s_{i2k}: i, k = 1, 2\}\}$, and investors in class 3 have the partition $P_3 \equiv \{\{s_{ij1}: i, j = 1, 2\}, \{s_{ij2}: i, j = 1, 2\}\}$. Thus, investors in class κ cannot distinguish states with the same κth index of the triple (i, j, k).

The entrepreneur receives a supernormal salary $\pi = 1$ and retains 0.5% of the equity (security 1), so that $\sigma = 0.005$. External investors are offered 99.5% of the equity for a total price of $p_1 = 410$. Security 2 is offered to external investors at a total price of $p_2 = 2600$. The rest of the relevant data are given in Table IIIA. The last two columns in this table are based on the columns for $v_1(s)$ and $v_2(s)$ and on the values of σ and π. Specifically, $v_E(s) = \sigma[v_1(s) + p_1]/(1 - \sigma) + \pi$, which follows from Eqs. (4) and (6), and $x_s - C = v_E(s) + v_1(s) + v_2(s)$, which follows from Eq. (7).

The hypothesized market demands for each security, $Q_n(s)$, and the implied values of $v_n(s)/Q_n(s)$ are shown in Table IIIB. Since there is

TABLE IIIA
THE VALUATION OF SECURITIES AND NET REVENUES, GIVEN s

s	$\rho(s)$	$v_1(s)$	$v_2(s)$	$v_E(s)$	$x_s - C$
s_{111}	0.2	−105	0	2.533	−102.467
s_{121}	0.1	−333	345	1.387	13.387
s_{112}	0.1	−300	270	1.553	−28.447
s_{122}	0.1	312	−303	4.628	13.628
s_{211}	0.1	−333	345	1.387	13.387
s_{221}	0.1	360	−330	4.869	34.869
s_{212}	0.1	312	−303	4.628	13.628
s_{222}	0.2	0	−105	3.060	−101.940

TABLE IIIB
HYPOTHESIZED MARKET DEMANDS AND VALUE OF SUBSCRIPTIONS

s	$Q_1(s)$	$v_1(s)/Q_1(s)$	$Q_2(s)$	$v_2(s)/Q_2(s)$
s_{111}	0	—	3	—
s_{121}	1	−333	2	172.5
s_{112}	1	−300	2	135
s_{122}	2	156	1	−303
s_{211}	1	−333	2	172.5
s_{221}	2	180	1	−330
s_{212}	2	156	1	−303
s_{222}	3	—	0	—

incomplete subscription if the state is s_{111} or s_{222}, the entries for $v_n(s)/Q_n(s)$ for these states are given by dashes.

The value to each class of investor of subscribing to each security, computed according to Eq. (10), is shown in Table IIIC.

Each investor subscribes to security n only when the expected value of subscribing to that security, $V_n(t_i, \xi)$, is non-negative. The implied security–demand correspondence is given in Table IIID.

The security–demand correspondence in Table IIID is consistent with the hypothesized market demands, $Q_n(s)$, in Table IIIB. Thus, the security–demand correspondence is part of a competitive security equilibrium. We defer an intuitive discussion of this equilibrium to the next section, where securities 1 and 2 are interpreted as stocks and bonds, respectively.

Three remarks are of interest at this point. First, there is one state, s_{112}, for which there is complete subscription despite the fact that all classes of investors believe the equilibrium is D-interim inefficient. Note also that in

TABLE IIIC
VALUE OF SUBSCRIBING TO EACH SECURITY

	Investor class 1		Investor class 2		Investor class 3	
s	$V_1(t_1, \xi)$	$V_2(t_1, \xi)$	$V_1(t_2, \xi)$	$V_2(t_2, \xi)$	$V_1(t_3, \xi)$	$V_2(t_3, \xi)$
s_{111}	−95.4	0.9	−95.4	0.9	−97.2	3
s_{121}	−95.4	0.9	0.6	−92.1	−97.2	3
s_{112}	−95.4	0.9	−95.4	0.9	2.4	−94.2
s_{122}	−95.4	0.9	0.6	−92.1	2.4	−94.2
s_{211}	0.6	−92.1	−95.4	0.9	−97.2	3
s_{221}	0.6	−92.1	0.6	−92.1	−97.2	3
s_{212}	0.6	−92.1	−95.4	0.9	2.4	−94.2
s_{222}	0.6	−92.1	0.6	−92.1	2.4	−94.2

TABLE IIID
SECURITY–DEMAND CORRESPONDENCE

s	Securities that class 1 investors subscribe to, $\xi(s, 1)$	Securities that class 2 investors subscribe to, $\xi(s, 2)$	Securities that class 3 investors subscribe to, $\xi(s, 3)$
s_{111}	{2}	{2}	{2}
s_{121}	{2}	{1}	{2}
s_{112}	{2}	{2}	{1}
s_{122}	{2}	{1}	{1}
s_{211}	{1}	{2}	{2}
s_{221}	{1}	{1}	{2}
s_{212}	{1}	{2}	{1}
s_{222}	{1}	{1}	{1}

this example, a security is rationed only for values of s for which the security has an ex post positive value.

Second, the financing scheme in this equilibrium dominates (for both society and the entrepreneur) single-security (100% equity) financing. A single-security equilibrium results in the project's being financed either when the state is s_{111} or when it is s_{222}, because s_{111} and s_{222} are in different elements of the partition for all investors. If any investor class buys the sole security when the state is s, then either s_{111} or s_{222} is in the element of that investor's partition containing s; thus the investor class will buy the security when the state is s_{111} or when the state is s_{222}. Thus, the project will be completely subscribed for either s_{111} or s_{222}. But if the project is financed for s_{111} or s_{222}, the equilibrium will be ex ante inefficient. By Theorem 1, there are no ex ante inefficient equilibria, and so the project would never be financed by 100% equity. However, under the financing scheme in this example, $\Sigma_{s\in D}\,(x_s - C)(\rho(s)/[\Sigma_{s'\in D}\,\rho(s')]) > 0$, so this financing scheme dominates 100% equity for society. It also dominates 100% equity for the entrepreneur, since $\Sigma_{s\in D}\,v_{\mathrm{E}}(s)(\rho(s)/[\Sigma_{s'\in D}\,\rho(s')]) > 0$.

Third, we have not ruled out the possibility of multiple competitive security equilibria. However, if there are other competitive security equilibria, they cannot be ex post efficient. The only security–demand correspondences consistent with ex post efficiency (described in the Appendix) are not consistent with equilibrium for this example.

VI. STOCKS AND BONDS

Up to this point we have examined security equilibria by specifying the values of $v_{\mathrm{E}}(s)$, and $v_n(s)$, $n = 1, \ldots, N$ for all states in S. Using the relation in Eq. (7), we examined the value of revenue minus costs without explicitly relating revenues x_s or payoff functions y_n to any underlying return streams, denoted z, generated by the project. In this section, we extend Example 3 by associating a distribution function F_s (with density f_s) to each state s that determines the return stream z, and we interpret security 1 as equity and security 2 as debt.

The entrepreneur raises capital equal to $p_1 + p_2$ by selling securities to external investors. After the entrepreneur receives a supernormal salary π and spends C to undertake the project, the available capital, denoted K, is

$$K = p_1 + p_2 - \pi - C. \tag{14}$$

After the project's return is realized, the total available resources, $K + z$, are distributed to the holders of securities (including the entrepreneur who holds a fraction σ of the equity).

Bonds promise to pay an aggregate amount R provided that the available funds, $K + z$, are sufficient to pay this amount. If the available funds are less than R, the bondholders take all that is available. Therefore, the payoff function to bondholders is

$$\tilde{y}_2(z) = \min[R, K + z], \tag{15}$$

and

$$y_2(s) = \int \tilde{y}_2(z) f_s(z) dz. \tag{16}$$

Thus, $y_2(s)$ is the expected payoff to bonds, evaluated using the distribution F_s.

The stockholders are the residual claimants. If the available resources exceed the amount that must be paid to bondholders, then the stockholders receive the excess, $K + z - R$. However, if the available funds are less than R, then the stockholders receive nothing. Therefore,

$$\tilde{y}_1(z) = \max[K + z - R, 0], \tag{17}$$

and

$$y_1(s) = \int \tilde{y}_1(z) f_s(z) dz. \tag{18}$$

Thus, $y_1(s)$ is the expected payoff to stocks, evaluated using the distribution F_s.

We suppose that z is distributed according to an exponential distribution. For each s, there is a pair of parameters $\alpha > 0$ and $\beta > 0$ determining the density function $f(z; \alpha, \beta) = (1/\beta) \exp[-(z - \alpha)/\beta]$ for $z > \alpha$, so that each $s \in S$ is identified with a pair (α, β). The mean and variance of revenues are $\mathrm{E}\{z\} = \alpha + \beta$ and $\mathrm{Var}\{z\} = \beta^2$. We can calculate the value of bonds to external investors by substituting (16), and the exponential density function into Eq. (5). Similarly, we can calculate the value of stock to external investors and to the entrepreneur by substituting (18) and the exponential density function into Eqs. (4) and (6). These calculations, which are straightforward but tedious, yield

$$v_1(\alpha, \beta) = (1 - \sigma)\beta \exp[-(R - K - \alpha)/\beta] - p_1, \tag{19}$$

$$v_2(\alpha, \beta) = K + \alpha + \beta - \beta \exp[-(R - K - \alpha)/\beta] - p_2, \tag{20}$$

and

$$v_{\mathrm{E}}(\alpha, \beta) = \sigma\beta \exp[-(R - K - \alpha)/\beta] + \pi. \tag{21}$$

In Table IIIE we show the values of α and β, that, when substituted into Eqs. (19)–(21), yield the values of $v_1(s)$, $v_2(s)$ and $v_{\mathrm{E}}(s)$ in Table IIIA. The characteristics of the securities in this case are $p_1 = 410$, $p_2 = 2600$, $R = 3110$, $\sigma = 0.005$, and $\pi = 1$, and the cost of the project is $C = 3000$. Thus, the security equilibrium that we presented earlier for the case with three classes of investors is an equilibrium when securities have these characteristics and the parameters of the distributions F_s are as given in Table IIIE.

Table IIIF presents, for each value of s, the conditional expectation $\mathrm{E}\{z \mid t_i(s) \cap D\}$ and the conditional standard deviation $\sqrt{\mathrm{Var}\{z \mid t_i(s) \cap D\}}$ of revenues for each investor class i, $i = 1, 2, 3$. We also show which security investor i subscribes to (each investor always subscribes to only one security).

TABLE IIIE
UNDERLYING EXPONENTIAL DISTRIBUTIONS WHEN $R = 3110$; $K = 9$; $p_1 = 410$; $p_2 = 2600$; $\sigma = 0.005$; AND $\pi = 1$

s	α	β	$E\{z\}$
s_{111}	1879.873	1017.660	2897.533
s_{121}	2727.570	285.817	3013.387
s_{112}	2559.972	411.581	2971.553
s_{122}	955.623	2058.005	3013.628
s_{211}	2727.570	285.817	3013.387
s_{221}	866.142	2168.728	3034.869
s_{212}	955.623	2058.005	3013.628
s_{222}	1589.983	1308.077	2898.060

TABLE IIIF
MEAN AND STANDARD DEVIATION OF z CONDITIONAL ON INVESTORS' INFORMATION AND COMPLETE SUBSCRIPTION

	Investor class 1			Investor class 2			Investor class 3		
s	Mean	S.d.	Security	Mean	S.d.	Security	Mean	S.d.	Security
s_{111}	2999.5	1223.1	bonds	2999.5	1223.1	bonds	3020.5	1273.7	bonds
s_{121}	2999.5	1223.1	bonds	3020.6	1734.0	stock	3020.5	1273.7	bonds
s_{112}	2999.5	1223.1	bonds	2999.5	1223.1	bonds	2999.6	1697.2	stock
s_{122}	2999.5	1223.1	bonds	3020.6	1734.0	stock	2999.6	1697.2	stock
s_{211}	3020.6	1734.0	stock	2999.5	1223.1	bonds	3020.5	1273.7	bonds
s_{221}	3020.6	1734.0	stock	3020.6	1734.0	stock	3020.5	1273.7	bonds
s_{212}	3020.6	1734.0	stock	2999.5	1223.1	bonds	2999.6	1697.2	stock
s_{222}	3020.6	1734.0	stock	3020.6	1734.0	stock	2999.6	1697.2	stock

As explained in the previous section, all three investor classes believe the equilibrium is D-interim inefficient at s_{112} (the conditional mean of revenue $E\{z|t_i(s_{112}) \cap D\}$ is less than the project cost for all three investor classes). At s_{112}, all three investor classes have essentially the same conditional mean of z, yet some of the investors subscribe to bonds while others subscribe to stock. In particular, the investors with relatively low conditional standard deviations (investor classes 1 and 2) subscribe to bonds while investors with relatively high conditional standard deviations subscribe to stocks.

Why do investors with high conditional variance subscribe to stocks and investors with low conditional variance subscribe to bonds? Stocks provide high payoffs in the event of high revenues z, but the downside risk on stocks is limited; once $z + K$ falls below the debt repayment R, it does not matter to the stockholder how far it falls. More precisely, the payoff function for stocks in Eq. (17) is a convex function of z. Thus, a mean-preserving spread of z will increase the expected payoff to stockholders, $y_1(s)$. Therefore, investors who view the distribution as having a high variance will buy stock. By contrast, bondholders are penalized by low values of z but do not share in the good fortune if z turns out to be extremely high. More precisely, the payoff function for bonds in Eq. (15) is a concave function of z, and so a mean-preserving spread of z will decrease the expected payoff to bondholders, $y_2(s)$. Hence, investors who view the distribution of z as having a relatively small variance will find bonds attractive. Note also that the different characteristics of bonds and stocks (not only from each other, but also from z) allow investors to evaluate the profitability of these securities differently, depending upon their private information.

VII. CONCLUSION

We have shown that if investors have heterogeneous asymmetric information, then competitive security equilibria will sometimes finance a project that every investor thinks has negative expected net revenues. Since the value of subscribing to a security differs from the expected value of the project's net revenue, it is easy to construct examples in which an investor believes that the project has negative expected net revenues, but is nonetheless willing to invest because other investors will bear a disproportionate share of the losses. Our examples are surprising because they show that it is possible for *all* investors to simultaneously and consistently believe this. We presented three examples illustrating two different ways in which this can occur.

The first way is through rationing. When a security is oversubscribed, the available securities are allocated to subscribers according to a random rationing scheme. Because the probability of acquiring a security after subscribing to it differs in different contingencies, there is a wedge between the expected value of subscribing to a security and the expected value of the project's net revenues, even if there is only one type of security. This is illustrated simply in Example 1. Example 2 shows that it is possible for a project to be financed with only one type of security, even though all investors believe that the project has negative value, and also know that all other investors believe that the project has a negative value. Consistency of beliefs does place some structure on what can occur: Theorem 2 states it cannot be common knowledge, however, that the project (when undertaken) has negative value.

The second way arises if a project is financed by two or more different types of securities. Although the bundle of all securities is a claim on the project's net revenues, different types of securities are different state-contingent claims on a project's net revenues. In Example 3, different investors will subscribe to different securities and thus finance the project even though no single investor would want to hold a bundle of all of the project's securities. Two attractive features of this example are that the information structure is consistent with the common assumption that agents receive independent signals, and securities are only rationed in states in which the project's net revenue is positive. Section VI shows how these securities can be interpreted as stocks and bonds.

In this analysis, we have treated the terms of the securities offered as exogenously given, and we have simply asked whether these securities will be fully subscribed. In the example with three classes of investors, the entrepreneur received a supernormal profit and retained a small share of the equity, so that the entrepreneur profited from this security offering. Investors also profited from the security offering (in a D-interim conditional sense) because they willingly subscribed to the securities after receiving their private information. One next step in this research project is to study the design of the menu of offered securities and their characteristics, and a further step is to study the sequential versions of the model.

APPENDIX

Proof of Theorem 1. Suppose $((\pi, \sigma, (p_n, y_n)), \xi)$ is a competitive security equilibrium. Define $S_{in} \equiv \{s \in D(\xi)\colon V_n(t_i(s), \xi) \geq 0\}$. Now,

$$\begin{aligned} 0 &\leq \textstyle\sum_i \sum_{\{t_i : t_i \cap S_{in} \neq \varnothing\}} \sum_{s \in t_i \cap D} [v_n(s)/Q_n(s, \xi)]\rho(s) \\ &= \textstyle\sum_i \sum_{s \in S_{in}} [v_n(s)/Q_n(s, \xi)]\rho(s) \\ &= \textstyle\sum_{s \in D} \sum_{\{i : s \in S_{in}\}} [v_n(s)/Q_n(s, \xi)]\rho(s) \\ &= \textstyle\sum_{s \in D} v_n(s)\rho(s), \end{aligned}$$

where the first equality follows from an investors' information t_i being pairwise disjoint, the second from the assumption that $((\pi, \sigma, (p_n, y_n)), \xi)$ is a security equilibrium (so that $\forall s \in D$, $\exists i$ s.t. $s \in S_{in}$), and the third from $Q_n(s, \xi) = |\{i : s \in S_{in}\}|$. This concludes the proof, since $\sum_{s \in D} (x_s - C)\rho(s) = \sum_{s \in D} \rho(s)\{v_E(s) + \sum_n v_n(s)\} \geq 0$. ∎

Proof of Theorem 2. Denote by $\wedge P$ the finest common coarsening, or *meet*, of $\{P_1, \ldots, P_M\}$; that is, $\wedge P$ is the finest partition of S having the property that, for all i, if $t_i \in P_i$, there exists $t \in \wedge P$ with $t_i \subseteq t$. An event $E \subseteq S$ is *common knowledge at s* if E contains that member of $\wedge P$ containing s (Aumann, 1976).

Fix $t \in \wedge P$. Since the argument proving Theorem 1 is still correct when we replace D by $t \cap D$, we have that $\sum_{s \in t \cap D} (x_s - C)\rho(s) \geq 0$.

We now argue to a contradiction. So define $E \equiv \{s \in S$: all investors believe the project is D-interim inefficient$\}$ and suppose E is common knowledge at s. Then $s \in t(s) \subseteq E$ for some unique $t(s) \in \wedge P$. For fixed i, for all $s' \in t(s)$, since $s' \in E$,

$$\sum_{s'' \in t_i(s') \cap D} (x_{s''} - C)\rho(s'') < 0.$$

Since $t(s) \in \wedge P$, we have $t(s) = \cup\{t_i(s')\colon s' \in t(s)\}$. Hence, since $t_i(s') \neq t_i(s'')$ implies $t_i(s') \cap t_i(s'') = \varnothing$, we have

$$\sum_{s'' \in t(s) \cap D} (x_{s''} - C)\rho(s'') = \sum_{\{t_i : t_i \cap t(s) \neq \varnothing\}} \sum_{s'' \in t_i \cap D} (x_{s''} - C)\rho(s'') < 0,$$

a contradiction. ∎

PROPOSITION. *Consider the securities and state contingent values in Table IIIA. Suppose ξ is a security–demand correspondence such that* $D(\xi) = \{s_{121}, s_{122}, s_{211}, s_{221}, s_{212}\}$. *Then ξ is given by either*

s	Class 1	Class 2	Class 3
s_{111}	{2}	{2}	{2}
s_{121}	{2}	{1}	{2}
s_{112}	{2}	{2}	$\varnothing$
s_{122}	{2}	{1}	$\varnothing$
s_{211}	{1}	{2}	{2}
s_{221}	{1}	{1}	{2}
s_{212}	{1}	{2}	$\varnothing$
s_{222}	{1}	{1}	$\varnothing$

or the pattern obtained by switching securities 1 *and* 2.

Proof of Proposition. Suppose there is an equilibrium in which there is complete subscription for $s \in \{s_{121}, s_{122}, s_{211}, s_{221}, s_{212}\}$ only. Now, it is never the case that an investor demands both securities in the same state. If an investor did demand both securities there would be full financing for at least one of $s \in \{s_{111}, s_{112}, s_{222}\}$.

We proceed through a series of claims.

CLAIM 1. *Investors in class* 1 *or* 2 *subscribe to a security when* $s = s_{112}$.

Proof. Suppose neither class subscribed to either security when $s = s_{112}$. Complete subscription when $s = s_{121}$ implies that class 2 subscribes to one of the securities when $s \in \{s_{121}, s_{122}, s_{221}, s_{222}\}$, which we take to be security 1 w.l.o.g., and class 3 subscribes to 2 when $s \in \{s_{111}, s_{121}, s_{211}, s_{221}\}$. But when $s = s_{211}$, class 2 does not subscribe to any security, so class 1 subscribes to security 1 when $s \in \{s_{211}, s_{221}, s_{212}, s_{222}\}$. Now, when $s = s_{122}$, 2 subscribes to 1 and 1 does not subscribe to any securities, so that 3 subscribes to 2 when $s \in \{s_{112}, s_{122}, s_{212}, s_{222}\}$. But this implies that, at s_{222}, 1 subscribes to 1 and 3 subscribes to 2, yielding complete subscription, a contradiction. ∎

CLAIM 2. *The same security (which, w.l.o.g., we will take as security* 1 *from here on) is unsubscribed at* s_{111} *and* s_{112}.

Proof. Since s_{111} and s_{112} are in the same element of 1's partition, class 1 investors must subscribe to the same security in these two states. The same is true of class 2 investors. By claim 1, 1 or 2 subscribes to some security at s_{112} and so at s_{111}. The other security is therefore unsubscribed at s_{111} and s_{112} (otherwise there would be complete subscription at these states). ∎

CLAIM 3. *Class 3 investors never subscribe to security 1.*

Proof. Follows immediately from Claim 2, since that claim implies that class 3 investors do not subscribe to security 1 when $s \in \{s_{111}, s_{121}, s_{211}, s_{221}\}$ or $s \in \{s_{112}, s_{122}, s_{212}, s_{222}\}$. ■

CLAIM 4. *Investors in classes 1 and 2 subscribe to security 2 at s_{112}.*

Proof. Clearly if classes 1 and 2 subscribe to any security at s_{112}, it must be to the same one (otherwise we have complete subscription). So, suppose first that class 1 does not subscribe to any security at s_{112} (and so also at s_{111}, s_{121}, and s_{122}). Then, from claim 1, class 2 subscribes to security 2 when $s \in \{s_{111}, s_{112}, s_{211}, s_{212}\}$. Full financing at s_{212} implies that 1 subscribes to security 1, since 3 never subscribes to security 1. Full financing at s_{122} requires 3 to subscribe to security 2 (and 2 to subscribe to security 1). But then there is complete subscription at s_{222}, since 1 subscribes to security 1 and 3 subscribes to security 2, a contradiction.

Now suppose that class 2 does not subscribe to any security when $s \in \{s_{111}, s_{112}, s_{211}, s_{212}\}$. Then 1 subscribes to security 2 for $s \in \{s_{111}, s_{121}, s_{112}, s_{122}\}$, and 2 subscribes to security 1 when $s \in \{s_{121}, s_{122}, s_{221}, s_{222}\}$. Full financing at s_{212} requires 1 to subscribe to security 1 when $s \in \{s_{211}, s_{221}, s_{212}, s_{222}\}$ and 3 subscribe to 2 when $s \in \{s_{112}, s_{122}, s_{212}, s_{222}\}$. But this is a contradiction, since it implies complete subscription at s_{222}. ■

Now, since there is complete subscription at s_{121} and s_{122}, class 2 investors subscribe to security 1 when $s \in \{s_{121}, s_{122}, s_{221}, s_{222}\}$. Also, since there is complete subscription at s_{211} and s_{212}, class 1 investors subscribe to security 1 when $s \in \{s_{211}, s_{221}, s_{212}, s_{222}\}$. Complete subscription at s_{221} implies that class 3 investors subscribe to security 2 when $s \in \{s_{111}, s_{121}, s_{211}, s_{221}\}$. Because class 1 and 2 investors subscribe to security 1 at s_{222}, class 3 investors do not subscribe to security 2 at s_{222} (or else there would be complete subscription). Thus class 3 investors do not subscribe to any security when $s \in \{s_{112}, s_{122}, s_{212}, s_{222}\}$. ■

REFERENCES

ALLEN, F., MORRIS, S., AND POSTLEWAITE, A. (1993). Finite bubbles with short sales constraints and asymmetric information, *J. Econ. Theory* **61,** 206–229.

AUMANN, R. J. (1976). Agreeing to disagree, *Ann. Statist.* **4,** 1236–1239.

BANERJEE, A. (1992). A simple model of herd behavior, *Quart. J. Econ.* **107,** 797–817.

BIKHCHANDANI, S., HIRSHLEIFER, D., AND WELCH, I. (1992). A theory of fads, fashion, custom, and cultural change as informational cascades, *J. Polit. Econ.* **100,** 992–1026.

BINMORE, K., AND BRANDENBURGER, A. (1990). Common knowledge and game theory, *in* "Essays on the Foundations of Game Theory" (K. Binmore, Ed.). Blackwell: Cambridge, MA.

GEANAKOPLOS, J. (1988), Common knowledge, Bayesian learning, and market speculation with bounded rationality, mimeo, Yale University.

GEANAKOPLOS, J. (1992). Common knowledge, *J. Econ. Perspectives* **6,** 53–82.

GEANAKOPLOS, J., AND POLEMARCHAKIS, H. (1982). We can't disagree forever, *J. Econ. Theory* **28,** 192–200.

JACKSON, M., AND PECK, J. (1992). Speculation and price fluctuations with private, extrinsic signals, *J. Econ. Theory* **55,** 274–295.

MILGROM, P., AND STOKEY, N. (1982). Information, trade, and common knowledge, *J. Econ. Theory* **26,** 17–27.

SEBENIUS, J., AND GEANAKOPLOS, J. (1983). Don't bet on it: Contingent agreements with asymmetric information, *J. Amer. Statist. Assoc.* **78,** 424–426.

WELCH, I. (1992). Sequential sales, learning, and cascades, *J. Finance* **47,** 695–732.